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POSITIVE ROTARY PUMPS, COMPRESSORS & EXHAUSTERS

By B. PUGH, B.Sc., A.M.I.Mech.E., A.M.I.Mar.E.

Published by The Draughtsman Publishing Co., Ltd.,
96 St. George's Square, London, S.W. 1.

SESSION 1936-37.

Printed by Milne, Tannahill, & Methven, 12-14 Mill Street, Perth.

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FOREWORD.

Positive Rotary Pumps, while popular in the 19th century, were eclipsed by the rapid strides made in the development of the centrifugal pump. Now, due in no small measure to the development in chemical engineering, coupled with certain advantages peculiar to this type of pump, they are again coming into favour. Literature dealing with them is rather sparse and incomplete, so the author feels that a pamphlet of this nature to serve as an introduction for those draughtsmen who are meeting this type of pump for the first time, is justified.

An exhaustive treatment would obviously be impossible in so small a space, so attention has been devoted to those pumps which illustrate a general principle, basic types, or those which meet with a wide application.

Many positive rotary pumps are capable of being used in modified forms as rotary engines or displacement meters. Those whose interests lie in these directions may also find something of value in the following pages.

The author is indebted to those firms who have so willingly supplied particulars of their products.

POSITIVE ROTARY PUMPS, COMPRESSORS AND EXHAUSTERS.

B. Pugh, B.Sc., A.M.I.Mech.E., A.M.I.Mar.E.

INTRODUCTORY.

Positive pumps are those pumps in which once the fluid has entered the pump chamber it is bound to be forced out again. The simple plunger pump belongs to this class. Since a fixed volume is pumped every revolution, it follows that, neglecting variation of leakage with speed, the discharge is proportional to the speed.

Positive rotary pumps are far more compact than reciprocating pumps of the same capacity, and the majority have the further advantage that no suction or delivery valves are required. The action of pumping usually involves sliding friction or very small clearances between the rotors and the casing, so these pumps are, generally speaking, not suitable for pumping liquids having solid matter in suspension. They are, however, capable of pumping a wide range of liquids—for example, coal tar, oils, molasses, viscose, fats, caustic liquors, acids, milk, etc. The volumetric efficiency, that is, the ratio of the actual volume pumped per revolution to the displacement, varies with the liquid pumped and with the speed, and, of course, depends largely upon the accuracy of manufacture. When the pump is running, the suction and delivery pipes must always be full open. Except where special means of doing so are provided, the delivery should be varied by varying the speed proportionately. Alternatively, excess of delivery may be returned to the suction side by means of a bye-pass fitted with a loaded regulating valve. Most types can be steam-jacketed if required, and readily cleaned when used for edible fluids. Also, they can readily be made in acid-resisting metals, *e.g.* monel metal for fruit juices and certain acids. They are particularly suitable for pumping against low heads and where churning of the liquid being pumped must be avoided. Their cost compares favourably with that of the centrifugal pump, and, moreover, they can deal with many liquids beyond the scope of the centrifugal. Positive rotary pumps have a large variety of forms, and to classify them is not easy. The two outstanding types are those having two revolving displacers or one revolving displacer and one revolving drum, geared together, and those having an eccentrically placed drum carrying one or more sliding or hinged vanes, or rollers, revolving in a cylindrical casing.

Both these types are very old. A gear-wheel pump, which is an example of the first type, was described in Daniel Schwenter's *Deliciae Physico-mathematicae*, published in 1636, while examples of the second type were described in Ramelli's *Le Diverse et Artificiose Machine*, published in 1588. Reproductions from engravings in both these works can be seen in the Science Museum, South Kensington. It is very probable that these old pumps were merely mechanical curiosities and did not find a practical application owing to lack of accurate means of manufacture. Rotary pumps did not come into general use until the 19th century.

Two other, though far less common types, are the rotary reciprocating, which is really just a reciprocating pump in which the cylinders revolve around the shaft, and the cam type, in which the fluid is displaced by a revolving cam.

We shall now consider these types in further detail.

THE TWIN DISPLACER TYPE.

Examples of this type of pump are the two and three lobe cycloidal pumps (Figs. 1 and 2), the crescent pumps (Figs. 11 and 12), the gear-wheel pump (Fig. 15) and the drum pump (Fig. 16). The displacers rotate in opposite directions either by the one gearing with the other, or by being made to do so by means of external gearing. To prevent the passage of liquid back from delivery to suction the displacers must remain continuously in contact with each other and with the casing. It remains to determine the condition under which the displacers will remain in contact with each other during one revolution. Let us assume that the displacers are connected by two gear wheels, so that their relative angular velocity remains constant. The pitch circles are shown dotted in the various figures. The relative motion of the displacers is that of the gears, which would be the same if one gear were fixed while the other rotated around it. The instantaneous centre of the motion is therefore the pitch-point of the two gears.

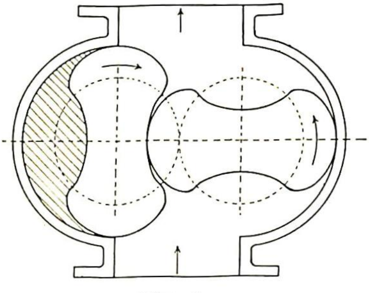


Fig. 1.

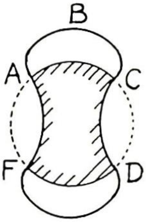


Fig. 1 (b).

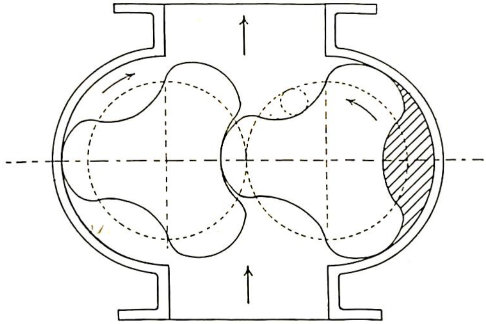


Fig. 2.

At this point of contact the two displacers have a common tangent. That is, the point of contact is instantaneously moving along the common tangent, and, since the pitch-point of the gears is the instantaneous centre of the motion, the condition to be satisfied in order that the two displacers remain in contact, *i.e.*, that throughout the motion they shall have a common tangent, is that the common normal to the two profiles must pass through the pitch-point of the two equal gears. If the driven displacer is capable of driving the other without the intervention of gears, these two pitch circles can still be drawn. When the two displacers rotate with the same speed the pitch circles will have diameters equal to the displacer centres, and the common normal to the profiles must pass through their point of contact, *i.e.* the mid-point of the displacer centres.

The two displacers, however, need not be equal in size. They may be so designed that one revolves faster than the other. In that case the two pitch circles will not have the same diameter, but it is easily seen that the same condition must be satisfied, that is, the common normal to the two profiles must always pass through the point of contact of the two pitch circles. The relative motion of the displacer contours will not always be pure rolling. In general, it will consist of rolling together with sliding. The velocity of sliding of the one over the other will, since they are moving in opposite directions, be given by the sum of their angular velocities multiplied by the length of the common normal.

If the shape of one displacer has been chosen, the shape of the mating displacer can be obtained by the construction of Fig. 3. Let the curve AB be a portion of the contour chosen. A number of points *a*, *b*, *c*, *d*, etc., are taken upon AB and the normals to AB drawn at these points. These normals will cut the pitch circle CD in points I, *k*, *l*, *m*, *n*, etc. On the pitch circle EF mark off arcs I*r*, I*s*, I*t*, etc., equal in length to the arcs I*k*, I*l*, I*m*, etc., and with radii *kb*, *lc*, *md*, etc., and centres *r*, *s*, *t*, etc., draw a series of arcs. This series of arcs will give the envelope of the mating curve GH, since when during rotation the point *r* coincides with *k*, the normal to GH will pass through (*rk*) and since the two curves have normals of equal length, *i.e.* (*k*, *b*) they will be in contact at *l*, similarly for other points on the two curves.

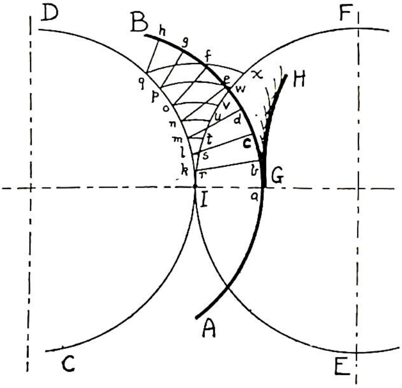


Fig. 3.

Alternatively the pitch circle EF can be drawn on a piece of drawing paper while the pitch circle CD, together with the curve AB, are drawn on tracing paper. With the aid of a pricker the pitch circle CD can be made to roll on the pitch circle EF and the successive positions of the curve AB traced through (see Fig. 4) will give the envelope of the required mating curve. This method is particularly useful in determining the profile of the double crescent displacer, the procedure in this case being illustrated by Fig. 13.

Any suitable displacer may be chosen and the shape of the mating displacer determined by one of the foregoing methods. It is usually more convenient to have two similar displacers. This

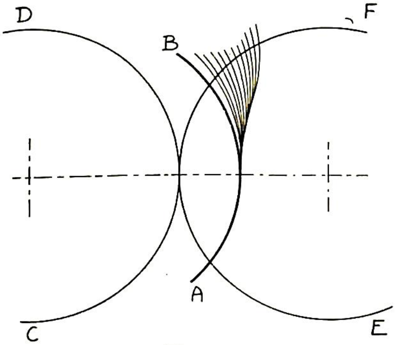


Fig. 4.

may be achieved by choosing the shape of each above the pitch circle and determining the shape below the pitch circle by a graphical method.

Now there are curves, the cycloidal curves and the involute, which, in virtue of the method by which they are generated, inherently possess the property that when they mate together their common normal passes through the pitch-point. Twin cycloidal displacers are in common use, the portion of the displacer above the pitch circle being epicycloidal, while that below is hypocycloidal. An epicycloid is the locus of a point on a circle known as the rolling circle, rolling on the outside of the pitch circle, while a hypocycloid is the locus of a point on the rolling circle when the rolling circle rolls inside the pitch circle. The same rolling circle is used to generate both the epicycloid and the hypocycloid. The epicycloidal portion of one rotor will be in contact with the hypocycloidal portion of the other.

That this state of affairs satisfies the necessary condition that the common normal passes through the point of contact of the pitch circles is seen from Fig. 5. The rolling circle when rolling on the upper pitch circle AB will generate the hypocycloid EF, while when rolling on the lower pitch circle CD it will generate the epicycloid GH. I is the point of contact of the pitch circles, and, since it is a point on both pitch circles, it is the point about which the generating point P is instantaneously rotating when describing either curve. That is, when describing both curves, the point P is instantaneously moving in a direction perpendicular to IP.

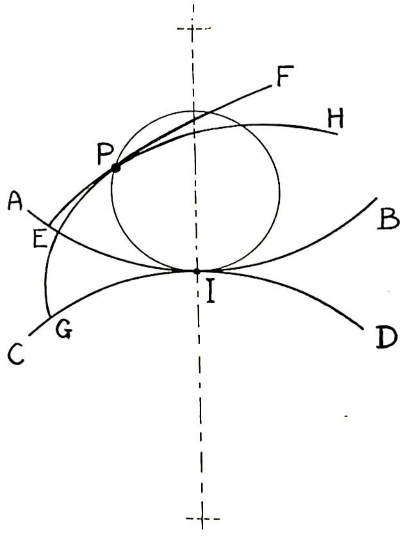


Fig. 5.

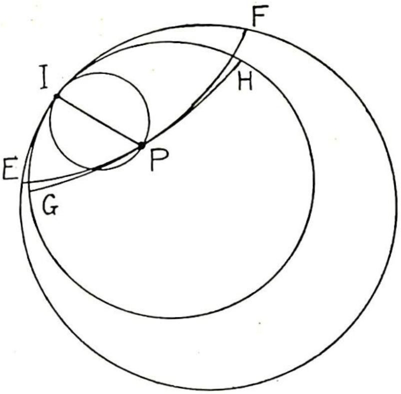


Fig. 6.

Hence the common normal PI passes through I, the point of contact of the two pitch circles. This shows that the rolling of an epicycloid upon a hypocycloid, and vice versa, satisfies the necessary condition. That it is also satisfied when one hypocycloid rolls upon another is easily seen from Fig. 6, in which the rolling circle when rolling on the outer pitch circle generates the hypocycloid EF, and when rolling on the inner the hypocycloid GH. This state of affairs is interesting in connection with "internal" pumps.

The involute can be looked upon as a special case of the epicycloid when the rolling has an infinite diameter. That is the curve generated by a point on a straight line when the line rolls upon a circle. A straight line cannot roll inside a circle, so it is considered to roll upon circles smaller in diameter than the pitch circles, known as base circles. Consequently the portions of the curve above and below the pitch circles are continuous curves. A normal to an involute will be a tangent to the base circle, consequently, when two involutes are in contact, it is easily seen that the common normal passes through the point of contact of the pitch circles.

An involute can also be looked upon as a curve generated by the end of a piece of string when the string is being unwound from the base circle.

The Epicycloid and Hypocycloid.

The geometrical methods of constructing these curves are not given here since they can be found in any good work on geometry. Fig. 7, however, has been reproduced for reference. (*a*) shows the state of affairs when the rolling circle diameter is one-half of the pitch circle diameter. The hypocycloid is then a diameter of the pitch circle. (*b*), (*c*) and (*d*) show the two curves when the rolling

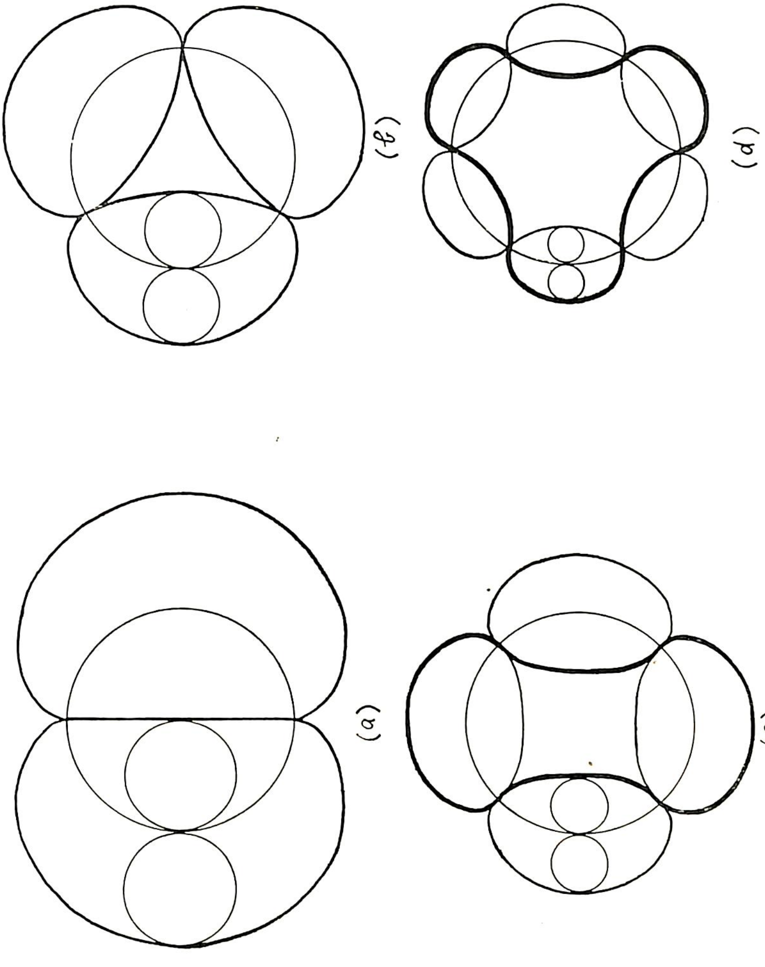


Fig. 7.

circle diameter is one-third, one-quarter and one-sixth of the pitch circle diameter respectively. The thickened line in (*c*) gives the profile of the Root's blower displacer, while that in (*d*) gives the shape of a three-lobe displacer. The diameter of the pitch circle must be a multiple of the rolling circle diameter.

It is obvious that the displacement per revolution of the two-lobe cycloidal pump will be twice the difference in area of the circumscribing circle and the displacer multiplied by the thickness of a displacer. To determine the displacement then, it becomes necessary to determine the area enclosed by the displacer profile. For a given size of displacer this may be obtained by means of a planimeter, or graphically, but, in order to obtain a general formula we shall employ the methods of the calculus. The same method may be applied to calculate the displacement of a cycloidal pump in which the rotors have any number of lobes. Firstly, it is necessary to obtain the general equations of the epicycloid and hypocycloid.

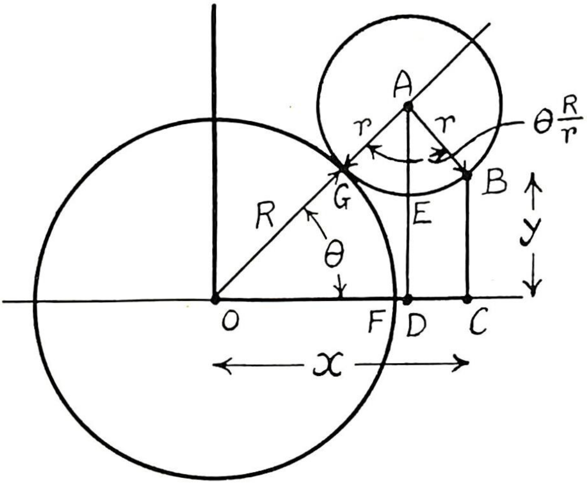


Fig. 8.

Referring to Fig. 8, B is the position of the tracing point when the rolling circle has rolled from F to G. Arc GB = arc GF, hence angle BAG = $\theta \frac{R}{r}$, R being the radius of the pitch circle and r the radius of the rolling circle.

$$\text{Angle DAB} = \theta \frac{R}{r} - 90^\circ + \theta$$

$$= \theta \left(\frac{R + r}{r}\right) - 90^\circ$$

and angle ABE $= 90^\circ - \left(\frac{\theta\,(R+r)}{r} - 90^\circ\right)$

$$= 180^\circ - \frac{\theta\,(R+r)}{r}$$

Hence $x = (R+r)\cos\theta + r\cos\left(180^\circ - \frac{\theta\,(R+r)}{r}\right)$

$$= (R+r)\cos\theta - r\cos\frac{R+r}{r}\theta \qquad (1)$$

$$y = (R+r)\sin\theta - r\sin\left(180^\circ - \frac{R+r}{r}\theta\right)$$

$$= (R+r)\sin\theta - r\sin\frac{R+r}{r}\theta \qquad (2)$$

giving the co-ordinates of B in terms of the angle θ.

When $R = 4r$ as in the case of the two-lobe cycloidal displacer,

$$x = 5r\cos\theta - r\cos\frac{5r}{r}\theta$$

$$= 5r\cos\theta - r\cos 5\theta \qquad (3)$$

similarly $y = 5r\sin\theta - r\sin 5\theta$ (4)

In the case of the hypocycloid, the rolling circle rolls inside the circle. The co-ordinates of the tracing point may now be found by changing the sign of the coefficient r in equations (1) and (2).

i.e., $x = (R-r)\cos\theta + r\cos\left(\frac{R-r}{r}\right)\theta$ (4)

$$y = (R-r)\sin\theta + r\sin\left(\frac{R-r}{r}\right)\theta \qquad (5)$$

when $R = 4r,$

$$x = 3r\cos\theta + r\cos 3\theta \qquad (5)$$

$$y = 3r\sin\theta + r\sin 3\theta \qquad (6)$$

from (5) $x = r\,(3\cos\theta + \cos 3\theta)$

$$= 4r\left(\tfrac{3}{4}\cos\theta + \frac{\cos 3\theta}{4}\right)$$

now $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

or $\cos^3\theta = \frac{\cos 3\theta}{4} + \frac{3\cos\theta}{4}$

hence $x = 4r\cos^3\theta$

and since $R = 4r$, $x = R\cos^3\theta$ (7)

similarly, from (6) it can be shown that

$$y = R\sin^3\theta \qquad (8)$$

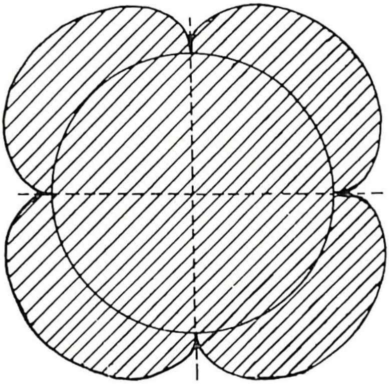


Fig. 9.

Considering first the epicycloid given by equations (5) and (6), see Fig. 9, the area enclosed by the curve is given by $\int y dx$. From Fig. 8 it will be seen that as θ increases the projection of the radius vector on the x axis decreases.

Hence $dx = -\dfrac{dx}{d\theta}\, d\,\theta$

Hence from (3), $dx = (5\,r \sin\theta - 5\,r \sin 5\,\theta)\; d\,\theta$

Hence the area enclosed by the curve

$$= \int_0^{2\pi} (5\,r \sin\theta - r \sin 5\,\theta)\;(5\,r \sin\theta - 5\,r \sin 5\,\theta)\; d\,\theta$$

$$= \int_0^{2\pi} 25\,r^2 \sin^2\theta - 5\,r^2 \sin\theta \sin 5\,\theta - 25\,r^2 \sin\theta \sin 5\,\theta + 5\,r^2 \sin^2 5\,\theta)\; d\,\theta$$

$$= \left[25\,r^2\left(\frac{\theta}{2} - \frac{\sin 2\,\theta}{4}\right) + 30\,r^2\left(\sin\frac{4\,\theta}{4} - \frac{\sin 6\,\theta}{6}\right) - 5\,r^2\,\frac{1}{5}\left(\frac{5\,\theta}{2} - \frac{\sin 10\,.\,\theta}{4}\right)\right]_0^{2\pi}$$

$$= 30\,\pi\,r^2$$

Since $r = \dfrac{R}{4},\; r^2 = \dfrac{R^2}{16}$

and $30\,\pi\,r^2 = \dfrac{30}{16}\,\pi\,R^2 = \dfrac{15}{8}\,\pi\,R^2$ (9)

Returning to the hypocycloid, the curve traced out by the point B is shown in Fig. 10, and we have now to determine the area shown shaded.

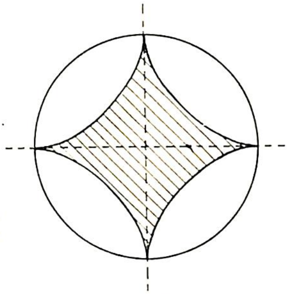


Fig. 10.

From (7) by differentiation,

$$dx = 3 \, R \cos^2 \theta \sin \theta \, d \theta$$

$$\int_{\theta=0}^{\theta=2\pi} y \, dx = \int_0^{2\pi} R \sin^3 \theta \, . \, 3 \, R \cos^2 \theta \sin \theta \, d \theta$$

$$= 3 \, R^2 \int_0^{2\pi} \sin^4 \theta \cos^2 \theta \, d \theta$$

$$= 3 \, R^2 \left[\frac{\theta}{16} - \frac{\sin^4 \theta}{64} - \frac{\sin^3 2 \theta}{48} \right]_0^{2\pi}$$

$$= \tfrac{3}{8} \pi R^2 \qquad (10)$$

Now one two-lobe cycloidal rotator consists of twice the area ABC + area ACDF (Fig. 1 (b)).

$$\text{Area ABC} = \frac{(1\tfrac{7}{8} - 1) \pi R^2}{4}$$

$$= \frac{7}{32} \pi R^2$$

$$\text{Area ACDF} = \pi R^2 - \frac{\pi R^2 - \tfrac{3}{8} \pi R^2}{2}$$

$$= \frac{11}{16} \pi R^2$$

Hence the total cross-sectional area of one rotator is

$$\frac{11}{16} \pi R^2 + 2 \frac{7}{32} \pi R^2$$

$$= 1\tfrac{1}{8} \pi R^2$$

The displacement of one rotator, *i.e.* twice the area cross-hatched in Fig. 1 (a)

$$= \pi \left(1\tfrac{1}{4}\right)^2 R^2 - 1\tfrac{1}{8}\,\pi R^2$$

$$= \frac{7}{16}\,\pi R^2$$

In an actual pump, of course, this area must be multiplied by the breadth of the rotator to give the displacement.

Hence, the displacement of a pump using two-lobe cycloidal displacers is

$$2 \,.\, \frac{7}{16}\,\pi R^2 \,.\, b$$

where R is the radius of the pitch circles and *b* the breadth of the displacers.

The actual quantity of liquid pumped per revolution will be

$$\tfrac{7}{8}\,\pi R^2 b \times \text{volumetric efficiency},$$

the volumetric efficiency depending upon the speed, head, and accuracy of workmanship.

The capacity of a pump employing two three-lobe cycloidal rotators can be determined in a similar manner, or, alternatively, a rotator can be accurately drawn and the displacement determined by means of a planimeter.

The two-lobe cycloidal displacer is commonly used for heads up to 30 ft. of water, while the three-lobe displacer pump (see Fig. 2) is used for heads up to 150 ft. The two-lobe pump will give four fluctuations in the delivery per revolution; the three-lobe, six and so on. Any number of lobes may be used, but, of course, the more lobes, the smaller is the delivery per revolution for a given size of pump chamber.

"Crescent" Pumps.

These form further examples of the twin displacer type. A pump using single crescent displacers is shown in Fig. 11. The portion AB could be involute in form, or cycloidal. If the portion

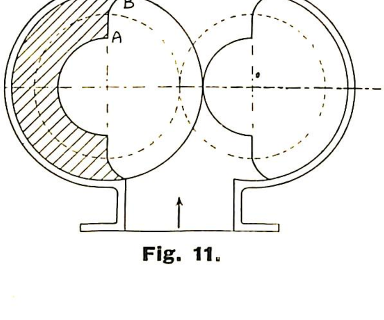


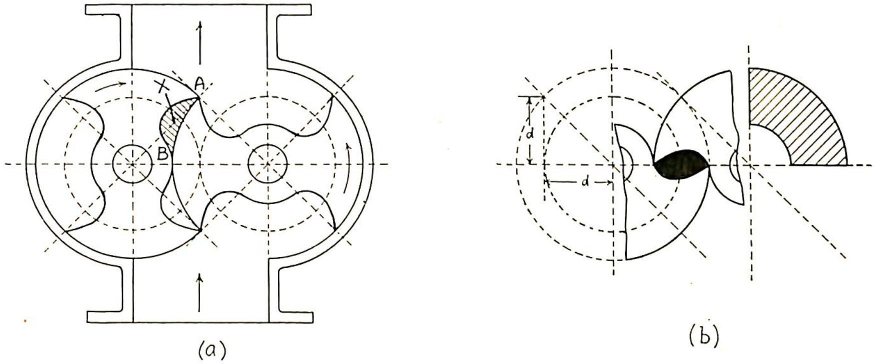
Fig. 11.

below the pitch circle is made a radial straight line, the portion above the pitch circle must be an epicycloid generated by a rolling circle having a diameter half that of the pitch circle.

The displacement will be twice the area shown shaded, multiplied by the width of a displacer, and there will be two pulsations per revolution in the discharge.

Double crescent rotors are used in the pump shown in Fig. 12. To ensure that during no part of the cycle is there a direct path from delivery to suction, the crescent must subtend an angle of 90° at the centre of the displacer. Then, if d is the diameter of the pitch circle, the outer diameter of the rotor $= 1{\cdot}414\, d$, and the diameter of the boss is

$$2\, d - 1{\cdot}414\, d = {\cdot}586\, d.$$



(a)

(b)

Fig. 12.

Hence the outer circles and the pitch circle can be drawn, and the point A determined. If the second pitch circle is drawn on a piece of tracing paper and the point A marked, this pitch circle can be made to roll upon the first. The locus of the point A will be found to be AB and thus the profile of the displacer is determined. The method will be clear from Fig. 13.

It will be seen that each revolution a volume equal to four times the blackened area (Fig. 12 [*b*]) multiplied by the breadth of the rotors is returned to the suction.

Hence the displacement (per revolution) is given by four times the cross-hatched area multiplied by the breadth of the displacers,

$$i.e.,\ b \left[\frac{\pi\,(1{\cdot}414\, d)^2}{4} - \frac{\pi\,({\cdot}586\, d^2)}{4}\right]$$

$$= b \left[\frac{\pi\,(2 - {\cdot}586^2)\, d^2}{4}\right] = 1{\cdot}3\, d^2\, b$$

But if D is the outside diameter of the rotors,

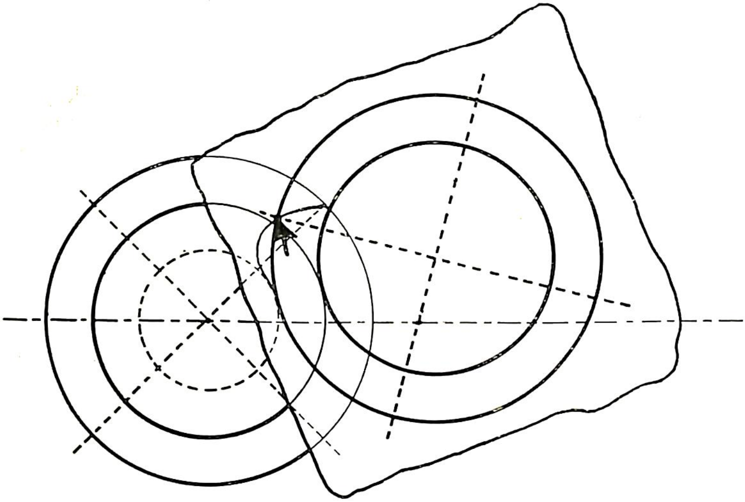


Fig. 13.

$$D = 1{\cdot}414\,d$$
$$D^2 = 2\,d^2$$

Hence, displacement $= \dfrac{1{\cdot}3\,b}{2}\,D^2 = {\cdot}65\,b\,D^2$

In the case of the two-lobe cycloidal pump we have seen that the displacement is given by $\frac{7}{8}\,\pi R^2 b$ where R is the pitch circle radius.

But $D = 2\tfrac{1}{2}\,R$

or $R^2 = \dfrac{D^2}{(2\frac{1}{2})^2}$

and the displacement $= \dfrac{7\,\pi}{8 \times 2\frac{1}{2} \times 2\frac{1}{2}}\,D^2\,b$

$= {\cdot}489\,b\,D^2$

So that for a given outside diameter and breadth of rotor the crescent shape rotor gives the greatest displacement.

Referring to Fig. 12, it will be seen that liquid will be trapped in the space X and then forced past the point A under great pressure. Consequently the action of pumping will be "harsh" and the pump will be noisy in operation. This can be obviated in two ways. The curve AB is really the "minimum" curve and more metal can be cut away if desired. If the arc AB be made longer there will be clearance between the horn of the engaging rotor and the one considered, enabling liquid which would otherwise be trapped to pass freely to the delivery.

Alternatively, the point A may be cut back so that the angle subtended by the crescent at the centre of the rotor is less than 90°. This, of course, may be done by making the outside diameter of the rotors greater than 1·414 times the pitch circle diameter, giving an increase in the displacement, but with an increase in the leakage loss. This leakage loss, however, will decrease with increase of speed and with increase in the viscosity of the fluid being pumped. The state of affairs when the outside diameter of the rotors is 1·5 times the pitch circle diameter is shown in Fig. 14 (*a* and *b*).

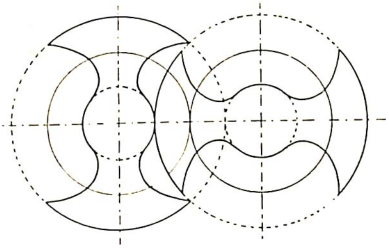


Fig. 14 (a).

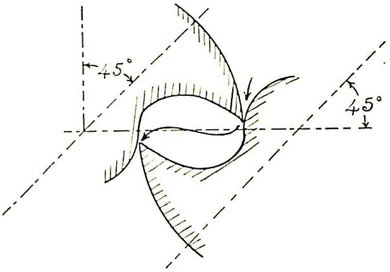


Fig. 14 (b).

The Gear-Wheel Pump.

This form of the twin displacer type of pump using gears with ordinary involute teeth provides a simple and cheap solution of the problem where a reliable pump of small size is required, and is in common use as lubricating pumps, suds pumps, etc. Its disadvantages are that its volumetric efficiency falls very rapidly as wear sets in, and even after a little wear it loses its suction lift, so that it becomes necessary to see that the pump is submerged wherever possible. Also, this form of pump is very easily jammed and delivers a rapidly pulsating flow. Its chief advantages lie in its simplicity and the fact that no external gearing is essential, though external gearing is sometimes used in heavy duty commercial gear pumps, to relieve the rotor teeth of some of the load.

Maximum backlash is necessary in order that fluid trapped in the spaces between engaging teeth may find a way out to the intake. To reduce the loss of liquid pumped to a minimum the total volume trapped between any two teeth should not vary, and this is ensured by making the time interval, reckoned in terms of the angular movement of the gears, during which the liquid is trapped, or, in other words, the time during which the two engaging teeth are making contact, as brief as possible. That is, the overlap of tooth action should be a minimum.

If spiral teeth are used, the spiral should be such that there is always a line of contact across the face width, otherwise there will be a direct passage from delivery to suction.

In order to obtain maximum discharge and maximum efficiency for a given centre distance and face width, the smallest practicable number of teeth must be used. When using ordinary involute teeth the minimum number of teeth is settled by the necessity of avoiding excessive undercutting. Using a 20° pressure angle the minimum number of teeth is twelve. Consequently pump manufacturers have aimed at obtaining a tooth profile which, by eliminating undercutting, permits a smaller number. Typical examples are the Albany pump and the Roloid pump. The internal construction of the Albany pump is shown in Fig. 15. It is made in sizes from $\frac{1}{4}$-in. giving 30 gallons per hour, to 7-in. giving 50,000 gallons per hour. The pump will draw from a depth of 25 ft. without priming, and is made to deliver against heads up to 800 ft.

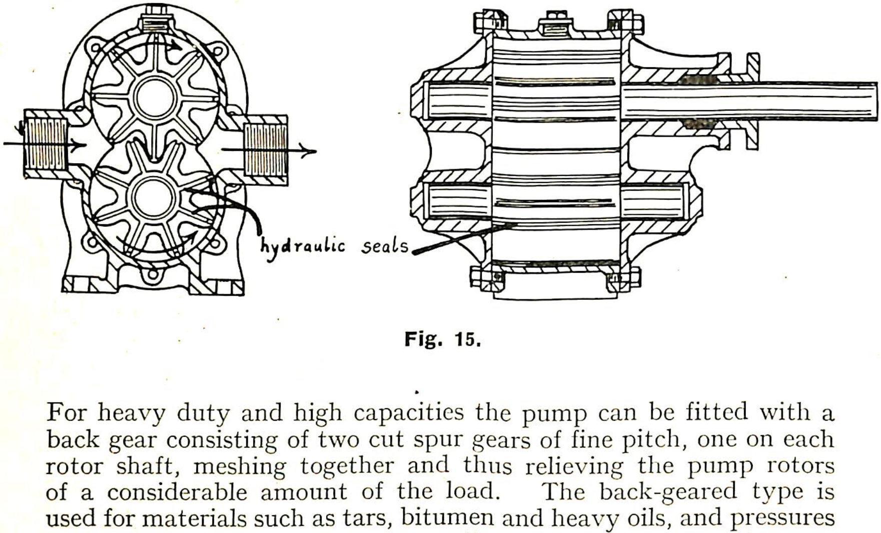


Fig. 15.

For heavy duty and high capacities the pump can be fitted with a back gear consisting of two cut spur gears of fine pitch, one on each rotor shaft, meshing together and thus relieving the pump rotors of a considerable amount of the load. The back-geared type is used for materials such as tars, bitumen and heavy oils, and pressures up to 200 lbs. per sq. inch can easily be obtained with this type. A special feature of this pump is the grooves cut on the face and edge of each tooth, thus providing hydraulic seals which retain the vacuum.

Roloid rotors are standardised with eight teeth of full standard depth. When used with lubricating oil at average speeds the volumetric efficiency lies between 90% and 95% and the overall efficiency between 40% and 60% according to pressure and viscosity. For heavy duty and high pressures, double helical gears having roloid profiles made of heat-treated carbon chromium steel shrunk on to nickel-chrome steel shafts are used. The possibility of helical leakage between the teeth is reduced to a minimum by lapping together the rotors at the correct centre distance.

Design of a Gear Pump using Standard Involute Gears.

Since for various reasons it is often desired to make a pump using standard involute gears, the calculation of gear sizes will be dealt with here. The discharge of a gear pump will, of course, be given by the sum of the volumes of the spaces in both wheels.

If D = pitch diameter of the gears in inches,
b = face width in inches,
s = diametral pitch of teeth,
n = pump speed in R.P.M.

Then $1/s$ = tooth addendum (standard proportion) and $D + 2/s$ = outside diameter of each gear.

The volume of the annulus consisting of the teeth and spaces of one gear

$$= b\left[\frac{\pi}{4}\left(D + \frac{2}{s}\right)^2 - \frac{\pi}{4}\left(D - \frac{2}{s}\right)^2\right]$$

$$= \frac{2\,\pi\,D\,b}{s}$$

Assuming that the volume of the spaces is equal to that of the teeth, this gives the total volume of the spaces of the two gears, and therefore the quantity pumped per revolution.

Hence the discharge of the pump will be

$$\frac{2\,\pi\,D\,b\,n}{s} \text{ cubic ins. per minute.}$$

The volumetric efficiency of such a pump will be between 80% and 85% and the overall efficiency about 40%.

The delivery varies inversely as the pitch number of the teeth, hence this should be kept as low as possible, consistent with the avoidance of excessive undercutting of the teeth.

Example.—Required a gear pump to deliver 10,000 cubic inches per minute and run at a speed of 500 R.P.M., delivery pressure 50 lbs./in.^{2}.

Assuming a volumetric efficiency of 80%, required theoretical delivery

$$= \frac{10{,}000 \times 100}{80} = 12{,}500 \text{ cubic inches.}$$

$$\text{Hence } 12{,}500 = \frac{2\,\pi\,D\,b\,.\,500}{s}$$

$$\text{or} \quad b = \frac{25\,s}{2\,\pi D} = \frac{3{\cdot}98\,s}{D}$$

Taking $s = 4$ and $D = 4''$, $b = 3{\cdot}98''$.

Hence two wheels of 4″ pitch circle diameter, 4″ wide, having 16 teeth, pressure angle 20° will satisfy the requirements.

An alternative is $D = 5''$, $b = 3\frac{3}{16}''$, 20 teeth.

If P is the delivery pressure in lb./in.^{2}, V = volume of liquid pumped per minute in cubic inches, assuming an overall efficiency of 40%, the horse-power required to drive the pump will be given by

$$\frac{P V}{12 \times 33{,}000 \times 0{\cdot}4}$$

$$= \frac{50 \times 10{,}000}{12 \times 33{,}000 \times 0{\cdot}4} = \mathbf{3{\cdot}16}$$

Drum Pump.

This well-known pump is shown in Fig. 16. Such a pump may be looked upon as a gear-wheel pump in which, on one wheel, all the teeth except two have been removed. It has the advantage that the action of pump involves the minimum churning of the liquid pumped, also use can be made of the momentum of the suction column. External gearing is, of course, necessary. If

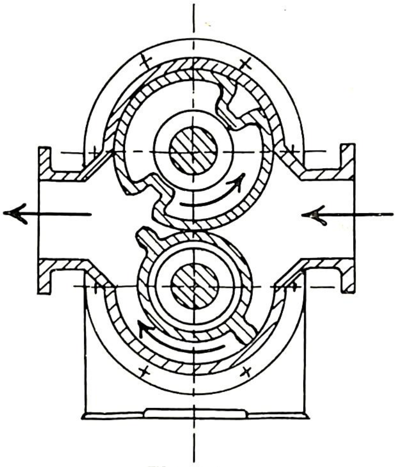


Fig. 16.

h is the height of the projecting rib, f its face width and D its mean diameter, the discharge of the pump at n R.P.M. is

$$\pi D f h n \times \text{volumetric efficiency.}$$

This pump is a good example of the type having one revolving displacer and one revolving sealing drum.

"Internal" Two-Displacer Pumps.

As has already been pointed out the two displacers may be made to rotate with unequal angular velocities. When the two displacers are not of the same size, a more compact pump results from placing one inside the other. If it is desired to use cycloidal displacers, then a two-lobe displacer can be used in conjunction with a three-lobe displacer having a pitch circle diameter of 3/2 times that of the two-lobe. A three-lobe displacer could be used with a four-lobe displacer having a pitch circle diameter 4/3 times that of the three-lobe. A four-lobe displacer could be used in conjunction with a six-lobe displacer when the ratio of the pitch circle diameters is 3/2 and so on. In general, however, when the difference in the number of lobes on the two rotors is greater than one, precautions have to be taken to ensure that, during no part of the cycle is there a direct path from delivery to suction.

We see, therefore, that the angular speeds of the two rotors are in the inverse ratio of the pitch circle diameters and the shapes of the rotors depend upon the choice of rolling circle. The shape of the inner displacer will be the locus of a point on the rolling circle as it rolls on the inner pitch circle while the shape of the outer will be the locus of a point on the rolling circle when it rolls on the outer pitch circle.

If the pitch circle diameter of the inner rotor is taken as two-thirds that of the outer and the rolling circle diameter half that of the smaller pitch circle, then the inner rotor becomes a straight line while the outer becomes a hypocycloid of three cusps.

This gives us the "two-three" pump of Fig. 17. In practice, of course, the inner displacer will become a rectangular bar with semi-circular ends, while the profile of the outer one will be parallel to the three-cusp hypocycloid. The action of pumping is shown in the following diagrams :—Fig. 17 (2) shows the commencement of suction through the port A, while (2), (3), (4), (5) and (6) shows various ports of the suction cycle through this port. (7) shows the completion of suction and the volume represented by the lines inclined at 45° is the total volume drawn in through the port A during one-half of a revolution of the outer rotor on three-quarter revolution of the inner. Suction through the port C has, however, commenced, so the total volume drawn in during three-quarter revolution of the inner rotor is represented by the cross-hatched area plus that covered by vertical lines beneath the internal rotor.

If the rolling circle is so chosen that its diameter is one-quarter that of the pitch circle of the inner rotor, then the inner rotor may be a two-lobe cycloidal rotor while the outer is of three-lobe formation. The action of pumping will be similar to that of the previous pump.

If the pitch circle of the inner rotor is made three-quarter the diameter of the outer, and the diameter of the rolling circle is made

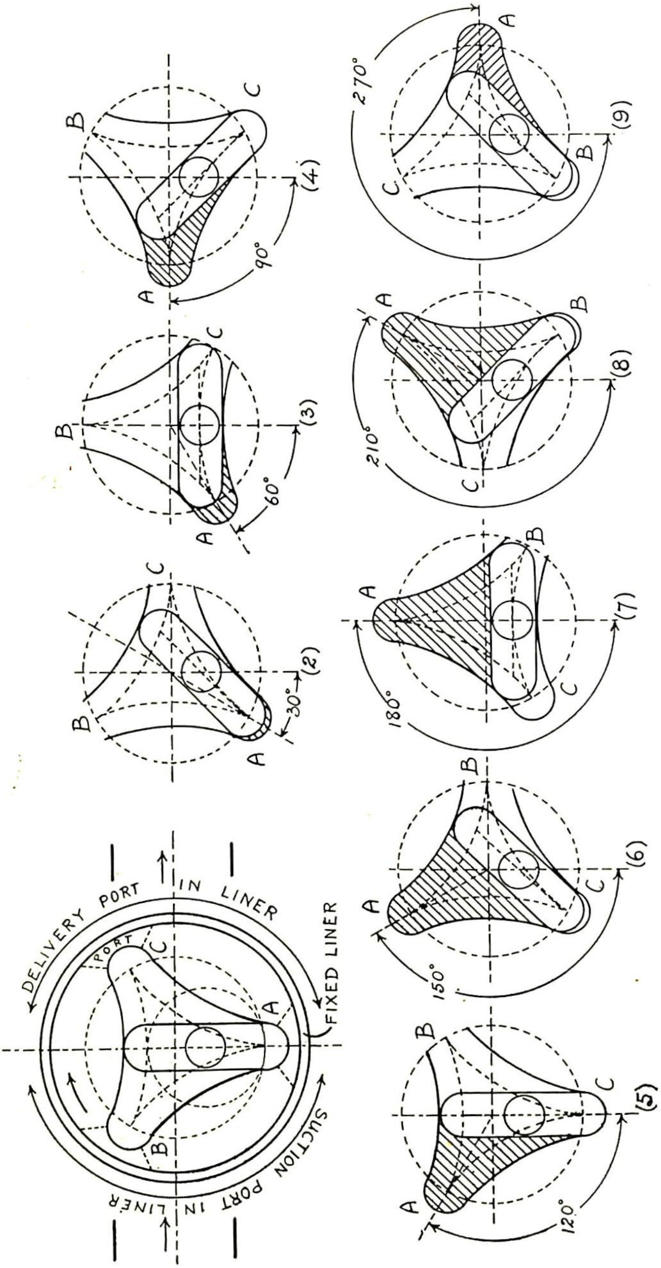

DELIVERY PORT IN LINER
FIXED LINER
SUCTION PORT IN LINER
PORT
A
B
C
30°
(2)
60°
(3)
90°
(4)
120°
(5)
150°
(6)
180°
(7)
210°
(8)
270°
(9)

Fig. 17.

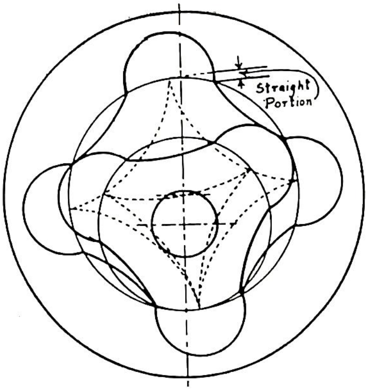


Fig. 18.

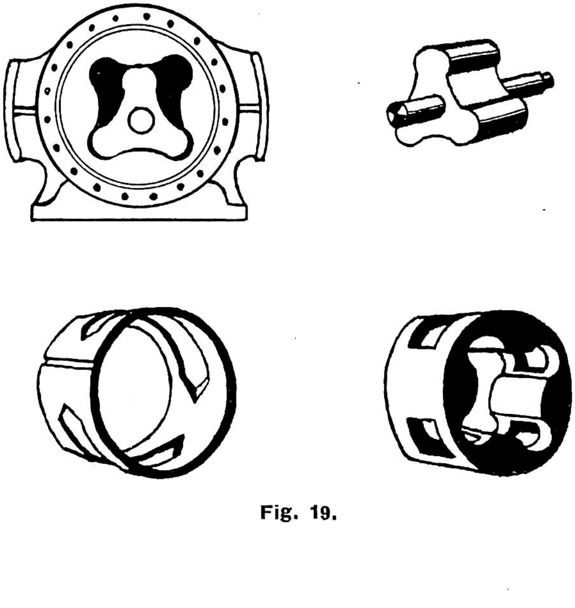


Fig. 19.

one-third the diameter of the inner rotor, then the inner rotor will be a three-cusp hypocycloid, while the outer rotor will be a hypocycloid of four cusps. To make these into rotors of practical form it is necessary to convert the points into portions of circles and make the rotor contours parallel to the original hypocycloids. Such a pump is shown in Fig. 18. The Feuerheerd pump is of this type. In this pump the inner rotor drives the outer which revolves freely in the casing. The four recesses in the outer rotor have short parallel sides and semi-circular ends. This form of lobe leads to slight variations in the velocity ratio but greatly facilitates manufacture. Constituent parts of this pump are shown in Fig. 19, while Fig. 20 shows the performance of this pump. A seven-eight type, in which the inner rotor has seven lobes while the outer has eight, is also made. This form is capable of being run at higher speeds than the three-four type and can usually be direct coupled to an electric motor without the use of gear transmission.

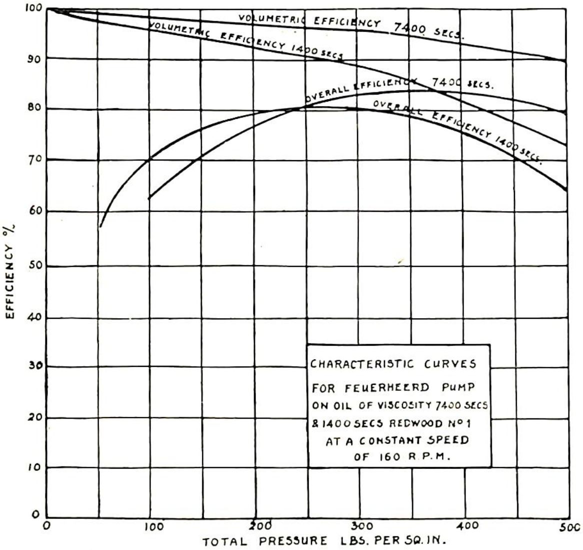


Fig. 20.

It has already been mentioned that it is not essential that the inner rotor have one lobe less than the outer. The Stone-Paramor pump, shown in Fig. 21, is an example, in which the inner rotor has four lobes while the outer has six. Here, however, it will be seen that a fixed crescent-shaped piece is necessary to prevent the return of fluid from delivery to suction. The inner rotor is connected to the driving shaft. The path of the fluid through the pump can easily be seen from the figure. These pumps have an

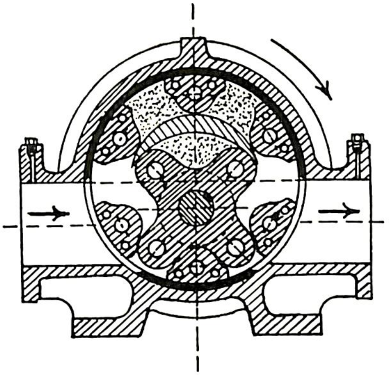


Fig. 21.

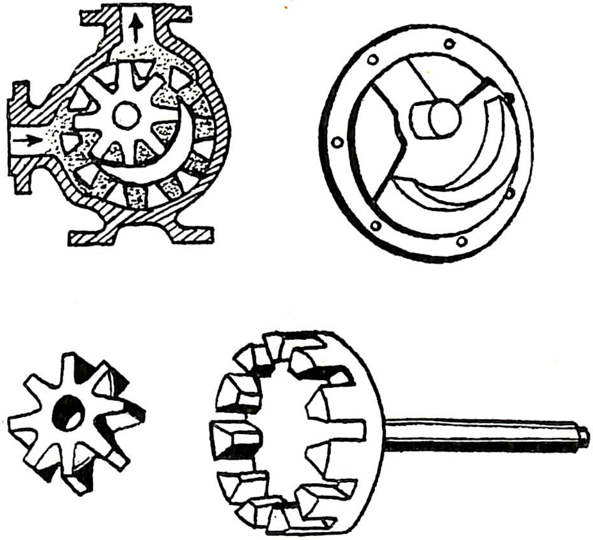


Fig. 22.

overall efficiency of round about 70% and an average volumetric efficiency of 95%.

An example of an internal gear pump is the "Rotan" pump, the constituent parts of which are shown in Fig. 22. In this case the outer rotor is driven while the inner is the "idler" and revolves on a pin in the cover. When the outer rotor is made to rotate in an anti-clockwise direction the "idler" moves away, causing a partial vacuum into which fluid from the suction is drawn. The path of the fluid through the pump can be followed from the figure.

An "internal" pump particularly interesting on account of its simplicity of construction is the Douglas pump shown in Fig. 23. The three pins are carried around by a face-plate in the direction of the arrow and in their progress rotate the drum which acts as a seal between suction and delivery. The drum is free to revolve on a pin in the cover. The crescent is fixed, being integral with the cover. The pins have holes drilled in them to allow the liquid to pass through as they move into and out of the slots. Fluid in the pump is shown cross-hatched in the figure. The pump is capable of a 20-25 ft. suction and a maximum total head of 80 ft. of water. For thick liquids an oversize pump is used at a reduced speed. With slight modification the pump can be used as a vacuum pump and is capable of maintaining a vacuum up to 27-28 inches of mercury.

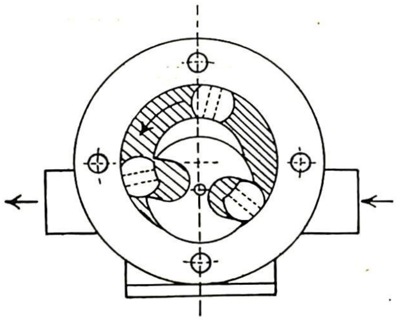


Fig. 23.

SLIDING VANE PUMPS.

Under this heading will be included all pumps having an eccentrically placed drum carrying one or more sliding vanes and rotating in a cylindrical casing. Pumps having hinged vanes, or rollers in place of vanes, may be considered as modifications. The simplest example is the Beale blower shown in Fig. 24, which is in common use for supplying air at low pressures. The two vanes are kept in contact with the casing by means of springs. Referring

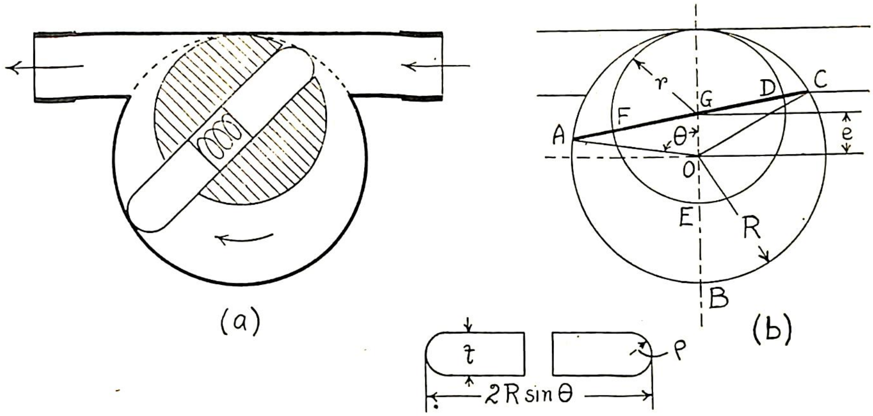


Fig. 24.

to Fig. 24 (*b*) and, for the moment, neglecting the blade thickness, it will be seen that during each half revolution of the drum a volume giving by area A B C D F, multiplied by the depth of the casing, is transferred from suction to delivery. If R is the radius of the casing, r the radius of the drum and e the eccentricity of the drum, then the displacement $=2\,d$ (sector AOCB + triangle GCO — semicircle FED), where d is the depth of the casing.

Hence displacement

$$= 2\,b\,[\,(\pi-\theta)\ R^2 + R\,e\ \sin\theta - \tfrac{1}{2}\,\pi r^2\,]$$

$$\text{where } \cos\theta = \frac{e}{R}$$

From this the volume occupied by the blades must be subtracted in order to obtain the correct displacement. If the blades have ends rounded to a radius ρ and have a thickness t, this volume is approximately given by

$$b\,t\,(2\,R\sin\theta - 2\,\rho) + \pi\rho^2 - 2\,r\,t.$$

Another pump of the sliding vane type is the "Essex" pump, shown with the cover removed in Fig. 25. This pump has no springs, the "throw" being given to the vanes by a pin. It has been found in practice that the vanes rarely touch the pin, centrifugal action keeping them in contact with the casing. The smaller sizes of this pump have two vanes. This pump is capable of a suction lift of 20-25 ft. and can be used for heads up to 100 ft. The displacement will be given by four times the volume enclosed between two adjacent blades, as the rear one cuts off the suction port.

The "Centrivac" pump is very similar, except that in this pump the vanes are held completely away from the casing by the hardened steel centre pin fixed in the cover. Hence there is always clearance

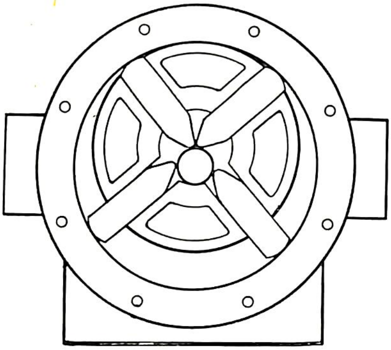


Fig. 25.

between the vane tips and the casing, and excessive friction at these points is eliminated. The drum also revolves clear of the casing.

The multi-vane form of the sliding vane pump is used only as an air compressor, and not as a pump for liquids, so it is discussed under "Compressors and Exhausters." A classical example of a pump having hinged vanes is the Lemeille ventilator. This machine is dealt with under the section on ventilators.

The Colebrook pump (Fig. 26) is an example of a pump in which vanes were replaced by cylinders. Centrifugal force is depended upon to keep the rollers in contact with the casing. This form is better adapted than the others for pumping liquids having solid matter in suspension. The displacement of such a pump is given by four times the volume enclosed between two adjacent rollers when the rear one cuts off the suction port. As far as the author is aware, the manufacture of this pump has been discontinued.

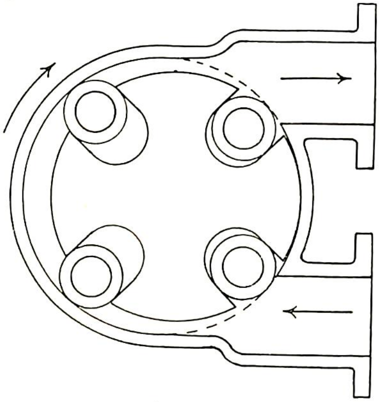


Fig. 26

An interesting offspring of the sliding vane pump is the "Birotor" pump, shown diagrammatically in Fig. 27, (*a*), (*b*), (*c*) and (*d*) showing successive positions of the moving parts. It may be looked upon as a vane pump in which the vane is fixed to the "casing" while the "casing" revolves with the drum, the outer container being necessary to surround the "casing" or outer rotor with liquid. The outer rotor is driven and drives the inner one by means of the vane which engages in a bushed slot. During one revolution the vane sweeps through the crescent-shaped space between the two rotors and forces the liquid through a port in the inner rotor and thence through the centre of the hollow fixed shaft. To overcome the sinusoidal variation in the rate of flow, the pumps are usually made with two or more units, working in parallel, with vanes disposed to each other in sub-multiples of 360°. The volumetric efficiency of this pump is normally about 97-98% and its overall efficiency 70%. It is capable of maintaining a suction lift of 25 ft. when dealing with non-volatile liquids.

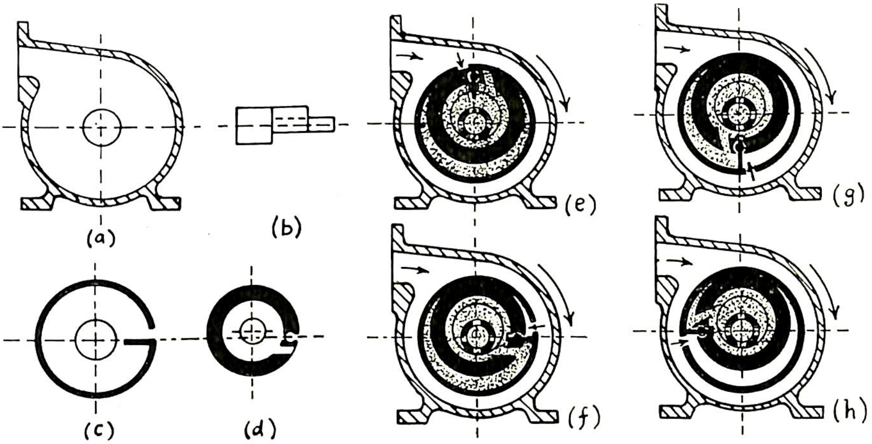


Fig. 27.

CAM PUMPS.

An example of a straightforward cam pump is the Mepstead pump shown in Fig. 28. The rotating piston is formed as a tee head on a disc keyed to the driving shaft and works in an annular space bounded on the outside by the casing and on the inside by bosses extending from the two end cover plates. The piston has sloping side faces which form tangents to the periphery of the supporting disc. The piston lifts the sliding abutment at every revolution. It will be seen that when the rotating piston is at the top of its path there is a direct path from delivery to suction. Consequently suction and delivery valves are necessary, and this is a serious drawback to what would otherwise be a very simple pump.

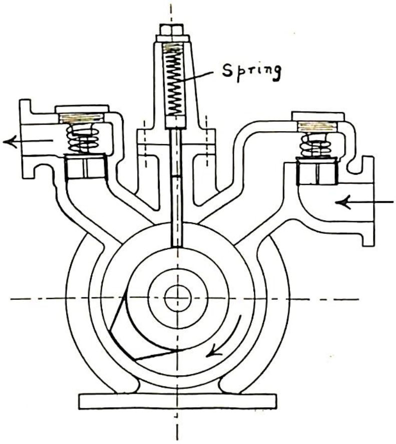


Fig. 28.

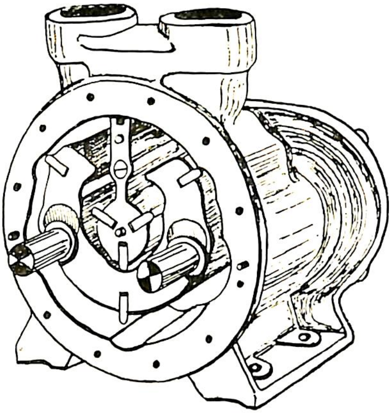


Fig. 29

The Hyvac pump shown in Fig. 36 is another example of a straightforward cam pump, but as this machine is only used as a vacuum pump it is dealt with under the section on exhausters.

The Varley paracyclic pump shown in Fig. 29 may be looked upon as a pump in which a hollow cam is employed in conjunction with a fixed abutment. Two cranks geared together give the displacer a motion such that every point in it moves in a circle of equal radius (paracyclic motion). Contact between the displacer and the casing is made by a number of spring-loaded contact strips having a movement in their slots only just sufficient to take up wear. The

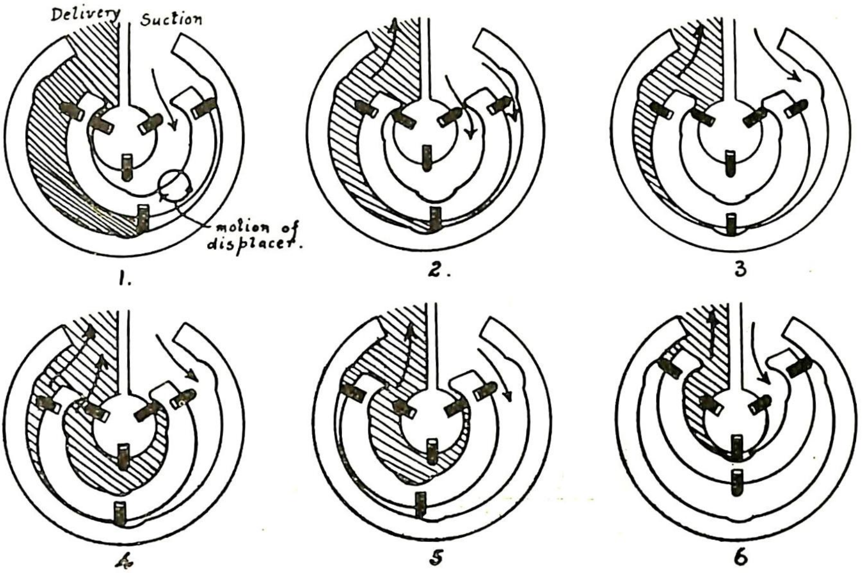


Fig. 30.

action of pumping can be followed from Fig. 30, which shows the displacer in six successive positions, the cross-hatching showing liquid under pressure.

In the Malcolm-Feuerheerd pump (Fig. 31) a number of circular cams move round an annular chamber formed between the casing and the hub at the same time rotating on eccentric pins. The pumping action is due to the relative approach and recession of adjacent cams. The eccentric pins upon which the cams or "roller pistons" rotate are connected to a revolving disc rotated by the

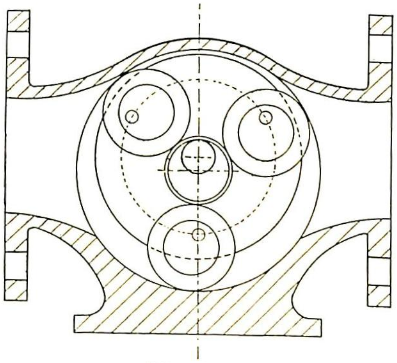


Fig. 31.

driving shaft. The roller pistons are surrounded by loose sleeves, while for pumping gritty liquids the pistons and the whole of the interior of the pump may be covered with rubber or other resilient material. The capacity of the pump may be varied by adjusting the eccentricity of the casing relative to the driving shaft of the piston carrier disc. The motion of a roller will be better understood if it is appreciated that the casing, rotating disc, roller, and its eccentric mounting may be replaced kinematically by a four-bar chain. The relative positions of any two rollers may then be found by the construction of Fig. 32.

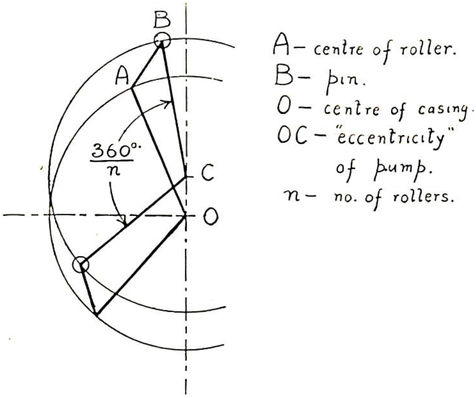


Fig. 32.

ROTARY RECIPROCATING PUMPS.

The chief advantage of this type of pump is the ease with which the delivery can be varied without altering the speed. These pumps can also be arranged so that the delivery is cut down to zero automatically when the pressure exceeds a predetermined amount. With pumps so arranged a relief bye-pass to suction becomes unnecessary.

There are two main types, those in which the pistons are disposed radially, and those in which the pistons are arranged parallel to the pump axis. An example of the first type is the Rotoplunge pump, while an example of the second is the swash-plate pump.

The simpler form of the Rotoplunge pump is shown in Fig. 33 (*a* and *b*). The pump cover carries an eccentric pin on which rotates the centre block. When the driving shaft is rotated the rotor carries round the plungers, but the eccentric pin and block causes the plungers to reciprocate in the rotor cylinders while at the same time each plunger slides over a face of the block. Consequently the liquid is drawn in by a plunger during one-half of a revolution, trapped between the shoe dividing the suction from the delivery side, and is expelled during the next half revolution. The larger sizes are fitted with side suction and yoke type plungers as shown in Fig. 33 (*c*), but the principle of operation remains the same. The direction of flow through the pump can be reversed without altering the direction of rotation by turning the cover carrying the eccentric pin through 180°. When it is often necessary to be able to change the direction of flow without changing the direction of rotation a cover free to rotate through 180° is fitted inside the actual pump cover. The stroke, and consequently the discharge of the pump, can be varied if required by means of a double eccentric motion capable of enabling the pin to travel in a straight line from the dead centre to the end of its stroke. In such a way the discharge can be varied automatically when the pumps are required for such purposes as oil fuel service.

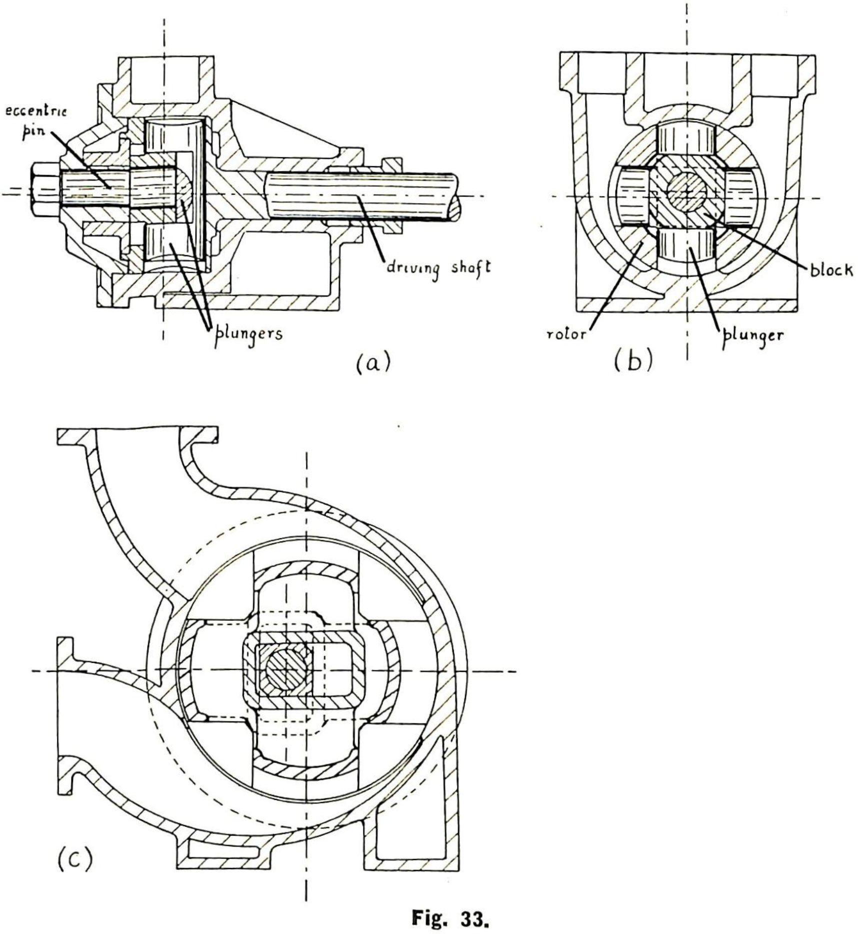


Fig. 33.

The swash-plate pump is shown diagrammatically in Fig. 34. The cylinder block has a number of cylinders bored in it parallel to the shaft and the block revolves with the shaft. The pistons are connected by ball-ended rods to the swash-plate which is kept in contact with the "tilting box." The swash-plate is connected to the shaft by a universal joint, while the tilting-box in which it revolves is carried on pivots. As the shaft revolves, the swash-plate wobbles, thus imparting reciprocating motion to the pistons. The stroke of the pistons depends upon the angle of tilt of the tilting-box and can be varied from zero, when the tilting box is perpendicular to the shaft, to a maximum by means of levers, or otherwise, and the direction of flow of the fluid can be reversed by reversing the angle of tilt of the tilting-box. The tilting-box should contain some means of taking both the thrust and the radial load upon the swash-plate. The cylinder block rotates in contact with the stationary valve plate which has suction and delivery ports cut in it as shown in the figure.

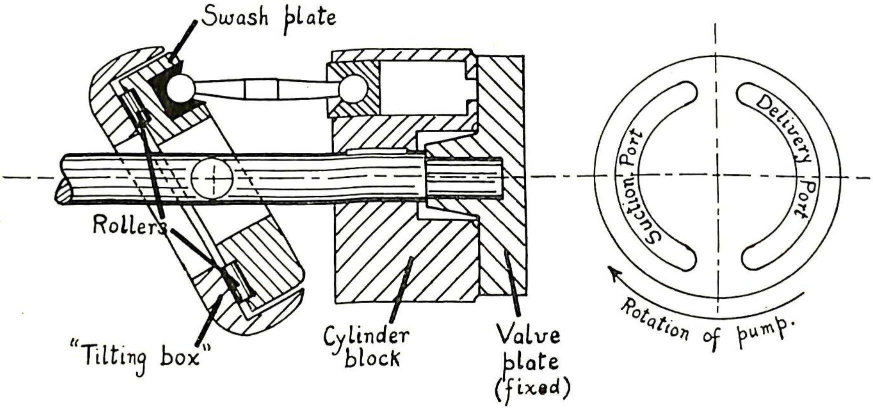


Fig. 34:

This type of pump is used in the Williams-Janney variable speed gear. In this gear a swash-plate pump is connected to a motor of similar construction. The pump is driven at constant speed and its discharge varied by varying the angle of the swash-plate. The motor unit, however, has a fixed stroke, consequently its speed will depend upon the quantity delivered by the pump. Hence, by varying the stroke of the pump the motor may be made to run at any speed lying between zero and the pump speed in the forward or reverse direction. The fluid used is oil.

COMPRESSORS, EXHAUSTERS, BLOWERS AND VENTILATORS.

When the object of the pump is to increase the pressure on the high pressure side, the pump is termed a compressor, while when its object is to reduce the pressure on the low pressure side, it is known as an exhauster. Blowers and ventilators are used to produce rapidly moving currents, the pressure of the delivery, or the degree of vacuum produced, as the case may be, being of secondary importance.

Most rotary positive pumps can be used as blowers or exhausters without alteration. The two-lobe cycloidal pump (Root's blower) and the double crescent type (Thwaite's blower) are in common use in this connection and as compressors for low pressures of the order of 5 lbs. per sq. inch. The two-vane sliding vane type (Beale blower) is used for such purposes as supplying air-blast for blowpipes. At high speeds the larger sizes of the twin displacer type of pump become noisy in action due to the large inertia of the displacers and the teeth of the connecting gears become subject to high inertia stresses. Consequently modern rotary compressors are almost invariably of the sliding vane type. Also, in this type the crescent-shaped air space can be divided up by the vanes into any convenient number of cells, the pressure difference between adjacent cells being only a fraction of the pressure pumped against, and hence the leakage across the blade tips can be kept low.

This type of compressor is commonly known as the "cellular" or "crescent" type. For pressures of the order of 12 to 15 lbs. per sq. inch a three-bladed sliding vane machine may be used. For higher pressures a larger number of blades become necessary, while for pressures of the order of 50 to 120 lbs. per sq. inch a two-stage machine is essential. For pressures above about 15 lbs. per sq. inch compressors should be water-cooled. Approximately 5 gallons per B.H.P. is required by single stage machines, and 6 to 8 for two-stage. At the usual working speeds the centrifugal force acting on the blades is far more than is sufficient to keep the blades in contact with the casing. Unless something is done to counteract this, there will be excessive friction and consequent heavy wear at the blade tips and the mechanical efficiency will be low. The main difference between the various makes lies in the manner in which this difficulty is overcome.

In the "Broomwade" compressor a non-metallic material having a low coefficient of friction and specially treated to withstand high temperatures, is used for the blades. The life of these blades is much longer than that of the usual blades and replacements can be made easily and cheaply. In the "Holland S.L.M." and "Reavell" compressors floating rings are used near each end of the rotor and revolving in unison with it. These rings take the centrifugal thrust of the blades, restricting and controlling their pressure.

Several rotary pumps with slight modification make excellent exhausters, but the majority of exhausters used in industry are of the sliding vane type. The design of these exhausters is similar to that of the cellular type compressor, in fact, if a compressor of this type is arranged with suitable piping and stop-valves it can be operated either as a compressor or exhauster without even altering the direction of rotation. Where a high vacuum and high volumetric efficiency are required, a compound machine is used. Fig. 35 shows, for medium-sized machines, the comparative volumetric efficiencies of a single stage and a compound exhauster.

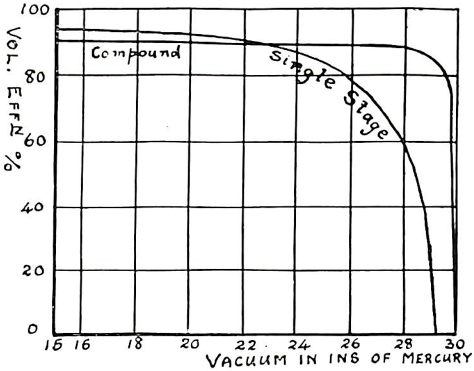


Fig. 35.

An exhauster in common use in laboratories, either alone or as a backing pump for mercury vacuum pumps, is the Hyvac pump shown in Fig. 36. This pump was referred to when cam pumps were dealt with. It is a straightforward cam pump, in which the abutment is kept in contact with the cam by means of a spring-loaded arm. In the outlet port is a ball non-return valve. Two units, working in series, are mounted side by side on a common shaft and the whole pump is immersed in oil.

An interesting machine that can be used either as a blower for smiths' fires, etc., or as an exhauster, is the Baker blower and exhauster, first introduced in 1873. Fig. 37 shows a section through this machine from which its working can be followed. All three shafts are connected by gearing, the two lower rotating at twice the speed of the upper or main shaft. The contour of the sealing drums can be found by rolling the pitch circle of the upper drum upon that of a lower when the tip of a blade will trace out the curve shown. The internal parts do not move in absolute contact, but the clearances are kept very small. Provision is made for drawing off such liquids as tar and their deposition on the inside of the machine cannot in any way injuriously affect its working or effi-

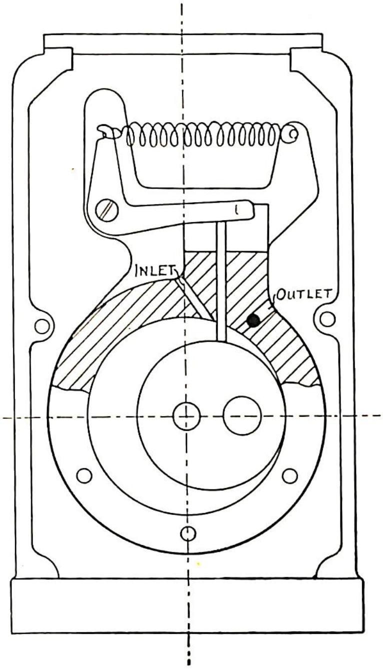


Fig. 36.

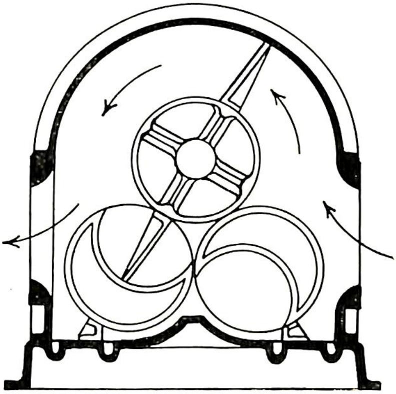


Fig. 37.

ciency. The interior can be inspected by removing the top cover without disturbing the inlet and outlet connections.

What is known as the "speed" of an exhauster is the volume of air or gas extracted per second from the low pressure side of a system, the volume being measured at the low pressure.

In dealing with exhausters it is sometimes necessary to know the time required by an exhauster of known capacity to reduce the pressure in a system to some lower value. The heating effect on the discharge stroke is to a large extent neutralised by the cooling effect that takes place on the suction side, so isothermal conditions may be assumed without appreciable error.

Let V be the volume of air or gas in the system.
v the displacement of the exhauster (per revolution).
P the absolute initial pressure in the system.
P_N the pressure in the system after the Nth revolution.

Then the pressure after the first revolution will be

$$P\left(\frac{V}{V+v}\right)$$

The pressure after the second revolution will be

$$P\left(\frac{V}{V+v}\right)\left(\frac{V}{V+v}\right)$$

and after the Nth revolution $P\left(\frac{V}{V+v}\right)^{N}$

$$\text{Hence } P_N = P\left(\frac{V}{V+v}\right)^{N} \qquad (1)$$

$$\text{Alternatively, from (1)} \quad \left(\frac{P_N}{P}\right)^{\frac{1}{N}} = \frac{V}{V+v}$$

$$\text{whence} \qquad v = \frac{V}{\left(\frac{P_n}{P}\right)^{\frac{1}{N}}} - V$$

giving the theoretical displacement of the exhauster in terms of the initial and final pressures and the volume of the system. The actual displacement of the exhauster will need to be greater than this by an amount depending upon the average volumetric efficiency of the exhauster during the operation.

Expressing (1) in logarithmic form we have

$$\log P_N - \log P = N \log\left(\frac{V}{V+v}\right)$$

whence $$N = \frac{\log P_N - \log P}{\log \left(\frac{V}{V + v}\right)}$$

or $$N = \frac{\log P - \log P_N}{\log (V + v) - \log V} \qquad (2)$$

giving the theoretical number of revolutions necessary, and hence, knowing the rotational speed of the exhauster, the theoretical time to reach a given low pressure in the system. The actual time, of course, also will depend upon the average volumetric efficiency of the exhauster during the operation.

An interesting result is obtained by assuming that the exhaustion follows a law of the form $P V^n = C$ where the exponent n depends upon the conditions.

The pressure at the end of the first revolution will be

$$P \left(\frac{V}{V + v}\right)^n$$

and at the end of N revolutions we have

$$P_N = P \left(\frac{V}{V + v}\right)^{Nn}$$

from which $$N n = \frac{\log P - \log P_n}{\log (V + v) - \log V}$$

or $$N = \frac{1}{n} \; \frac{\log P - \log P_n}{\log (V + v) - \log V}$$

from which, since n will always be greater than unity except for isothermal conditions when it is unity, it can be seen, by comparison with (1), that if the exhauster is of sufficient capacity to give the required reduction in pressure under isothermal conditions, all intermediate conditions between this and adiabatic can be met by a reduction in the rotational speed of the exhauster.

Although the fan has now driven out the positive type of ventilator, there are two machines originally introduced as ventilators which are of particular interest since their construction has formed the basis of many designs for pumps. These are the Fabry and Lemielle ventilators. They were introduced, the Fabry in 1850 and the Lemielle in 1853, to replace furnace draught in mines.

The original form of the Fabry ventilator is shown in Fig. 38, from which it will be seen that the machine is really a skeleton of the three-lobe cycloidal type of pump. Each displacer consists of three main arms to which floats are attached. To each main arm is fastened a cross arm, also carrying a float, and having epicycloidal

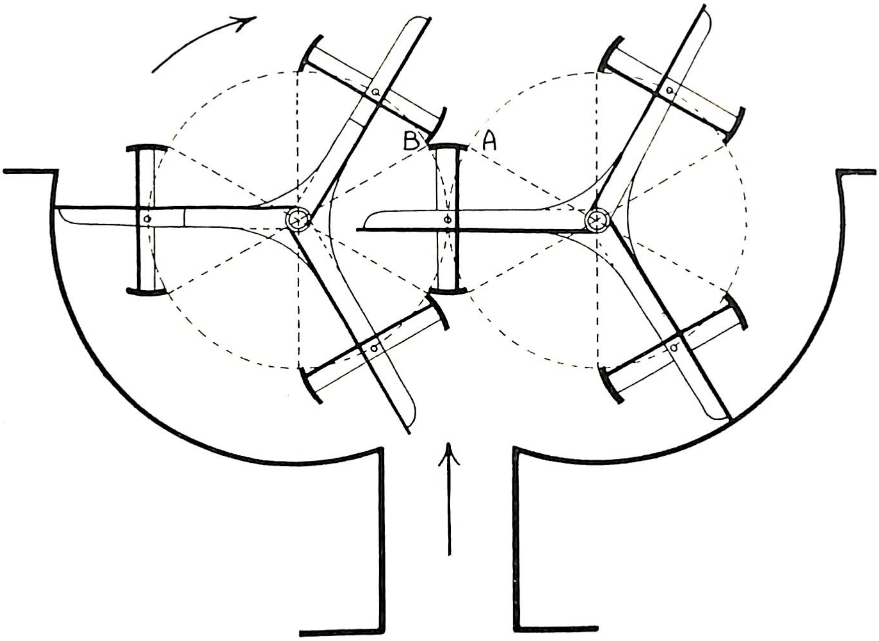


Fig. 38.

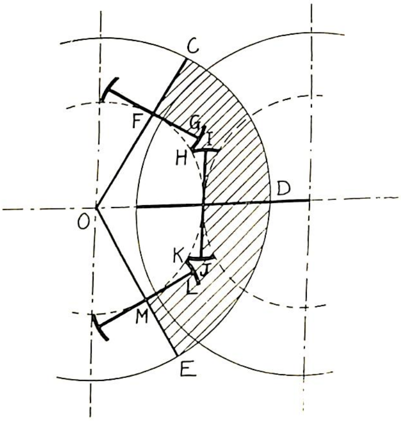


Fig. 39.

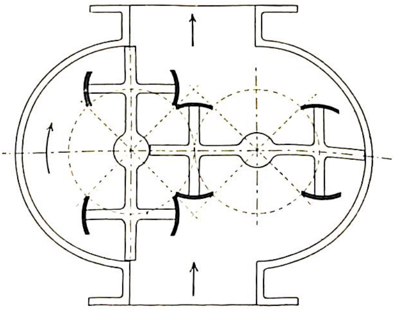


Fig. 40.

ends. The troughs in which the displacers hang were of masonry, lined with cement after the displacers had been hung in position. The displacers were connected by two equal gears and when they rotated the floats attached to the main arms drew in air from the mine and discharged it into the atmosphere. If the pitch circles are divided into six equal parts and the right-hand pitch-circle rolled upon the left-hand one, the point A will describe the path AB, giving the contour of the epicycloidal floats attached to the ends of each cross-arm.

In such a skeleton pump, two adjacent arms will enclose and carry round a volume of air equal to the sector OCDE (see Fig. 39) multiplied by the width of the floats. But a volume equal to the area O F G H I J K L M multiplied by the breadth of the floats will be returned to the suction. Consequently the displacement of such a pump will be six times the cross-hatched area of Fig. 39 multiplied by the width of the floats. The Fabry ventilators had a volumetric efficiency of about 70% and an overall efficiency of about 50%.

Later a two-arm type was introduced. This is shown in Fig. 40.

Lemeille's ventilator, shown in Fig. 41, consisted of a drum placed eccentrically in a masonry chamber and having attached to it a number of vanes pivoted at their base whose outer edges were kept close to the circumference of the chamber by connecting rods attached to a shaft fixed in the centre of the chamber. As the drum revolved, each vane in its turn passed the opening leading to the mine, thus trapping a volume of air between it and the preceding vane. This air then passed into the outlet, the discharge into the atmosphere being practically continuous.

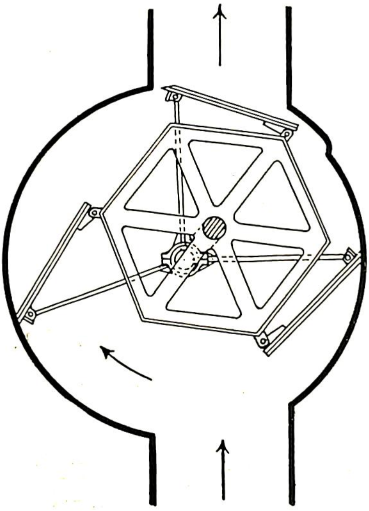


Fig. 41

Power absorbed by Compressors and Exhausters.

In the case of blowers and ventilators (or exhausters when the pressure difference is small) if Q is the quantity of air or gas dealt with in cubic feet per minute, p the pressure difference created in lbs. per square foot, then the horse-power required is given by

$$\frac{Q p}{33{,}000} \times \frac{100}{\text{Overall Efficiency (per cent.)}}$$

In the case of compressors, in practice the conditions are intermediate between isothermal and adiabatic, *i.e.* the compression follows a law of the form $PV^n = C$, an average value of n being 1·3. Since, however, when dealing with rotary compressors the isothermal efficiency is often given, we shall determine the work done when the gas or air is compressed isothermally, and when the law of compression is of the form $P\,V^n = C$.

Dealing first with isothermal compression (or expansion) and considering the case when the gas or air is compressed (or expanded) from a pressure and volume of P_1, V_1, to P_2, V_2, at any instant when the pressure is P let the volume change by a small amount dV. Then the work done during this small change in volume is

$$P\,d\,V = C\frac{d\,V}{V} \quad \text{since } PV = C.$$

Then the work done during the compression or expansion is

$$\int_{v1}^{v2} C\frac{d\,V}{V} = \Big[\,C \log_e V\,\Big]_{v1}^{v2} = C \log_e \frac{V_1}{V_2}$$

$$= P_1\,V_1 \log_e \frac{V_2}{V_1} \text{ and since } P_1\,V_1 = P_2\,V_2 = C$$

$$\text{this becomes } P_1\,V_1 \log_e \frac{P_2}{P_1}$$

Considering now compression or expansion according to the law $PV^n = C$, the work done will be

$$\int_{v1}^{v2} C\,\frac{d\,V}{V^n}$$

$$= \frac{C}{1-n} V_2^{1-n} - \frac{C}{1-n} V_1^{1-n}$$

$$\text{but } P_1\,V_1^n = P_2\,V_2^n = C$$

Hence the work done becomes

$$\frac{P_2\,V_2 - P_1\,V_1}{1-n} = \frac{P_1\,V_1 - P_2\,V_2}{n-1}$$

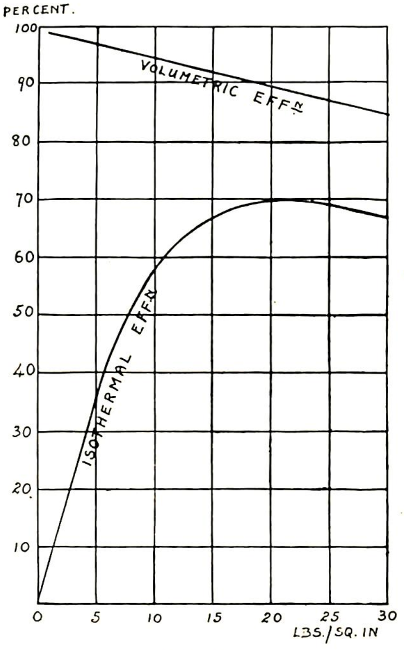

PERCENT.
100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
LBS./SQ. IN
VOLUMETRIC EFF.N
ISOTHERMAL EFF.N

Fig. 42.

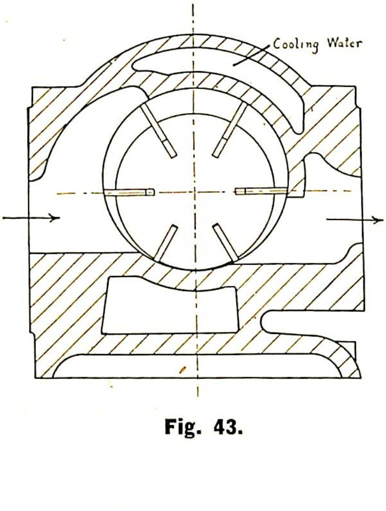

Cooling Water

Fig. 43.

P_1 and P_2 being in lbs. per sq. ft. and V_1 and V_2 in cubic feet if the work done is required in foot lbs.

When the air or gas is not compressed isothermally the temperature rises and more work is done in compressing the substance. Consequently isothermal compression is the ideal state of affairs. The "isothermal efficiency" is the ratio of the horse-power required to compress isothermally to the B.H.P. measured at the driving coupling. How this, together with the volumetric efficiency, varies for a medium size machine is shown in Fig. 42, compiled from results obtained on a compressor designed to deliver air at 20 lbs. per sq. in.

A section of a typical small sliding vane compressor having a six-bladed rotor is given in Fig. 43. The machine is also shown diagrammatically in Fig. 44 (*a*). An amount of air or gas of volume ABEF multiplied by the breadth of the blades, is drawn in. This is represented by V_1 in Fig. 44 (*b*) its pressure being P_1. The final volume is BCDE × vane breadth and is represented by V_2, its pressure being P_2. The process of compression can be represented by the curve XY, while the cross-hatched portion of the diagram represents the work done upon the air during compression.

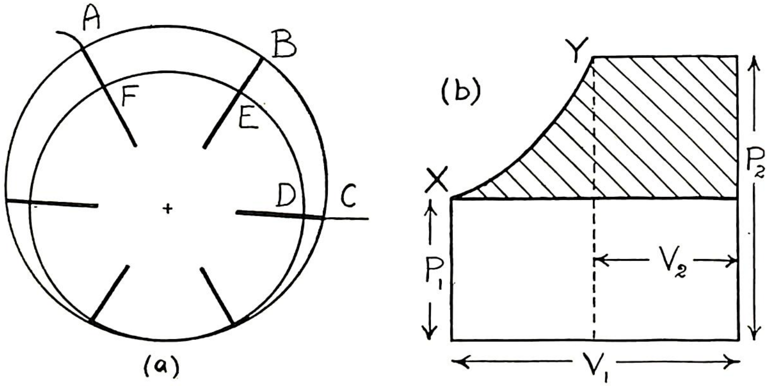


Fig. 44.

In the case of isothermal compression the work done during compression is given by

$$P_1 V_1 \log_e \frac{P_2}{P_1} + P_2 V_2 - P_1 V_1$$

and since $P_2 V_2 = P_1 V_1$ this becomes

$$P_1 V_1 \log_e \frac{P_2}{P_1}$$

If P_1 is the pressure of the atmosphere in lbs. per sq. ft.,
V_1 the volume of air compressed in cubic feet per minute,
P_2 the final pressure in lbs. per sq. ft. (absolute),

the horse-power required for isothermal compression will be

$$\frac{P_1 V_1 \log_e \frac{P_2}{P_1}}{33{,}000}$$

This, of course, is the ideal state. The horse-power required by an actual compressor will be

$$\frac{P_1 V_1}{33{,}000} \log_e \frac{P_2}{P_1} \times \frac{100}{\text{Isothermal Efficiency (per cent.)}}$$

Returning to Fig. 44 (*b*), in the case of compression according to the law $PV^n = C$, the work done

$$\frac{P_2 V_2 - P_1 V_1}{n-1} + P_2 V_2 - P_1 V_1$$

$$= \frac{n}{n-1} (P_2 V_2 - P_1 V_1)$$

and the efficiency of the compression is given by

$$\frac{P_1 V_1 \log_e \frac{P_2}{P_1}}{\frac{n}{n-1} (P_2 V_2 - P_1 V_1)}$$

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